Divergence-Conforming Collocation for the Incompressible Navier-Stokes Equations

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Background

- Collocation is a weighted residual method which requires the strong-form residual to vanish at a discrete set of points, called collocation points
- The enhanced global smoothness of B-spline basis functions enables the construction of well-defined collocation methods
- Isogeometric collocation methods have recently been explored in solid mechanics applications, where they can greatly decrease cost, but are less explored in fluids

Structure-Preserving B-spline Spaces

- Choosing the following spaces for discrete vorticity, velocity, and pressure can be shown to yield a method which exactly satisfies the continuity constraint:

$$\Psi_h := \{\boldsymbol{\psi}_h \in S^{p_1-1,p_2,p_3}_{\boldsymbol{\alpha_1-1,\alpha_2,\alpha_3}} \times S^{p_1,p_2-1,p_3}_{\boldsymbol{\alpha_1,\alpha_2-1,\alpha_3}} \times S^{p_1,p_2,p_3-1}_{\boldsymbol{\alpha_1,\alpha_2,\alpha_3-1}}\}$$

$$V_h := \{\mathbf{v}_h \in S^{p_1,p_2-1,p_3-1}_{\boldsymbol{\alpha_1,\alpha_2-1,\alpha_3-1}} \times S^{p_1-1,p_2,p_3-1}_{\boldsymbol{\alpha_1-1,\alpha_2,\alpha_3-1}} \times S^{p_1-1,p_2-1,p_3}_{\boldsymbol{\alpha_1-1,\alpha_2-1,\alpha_3}}\}$$

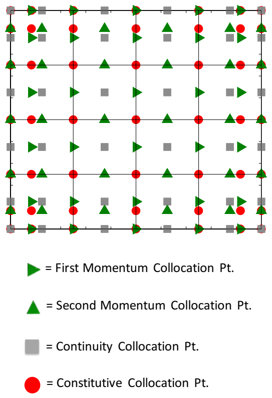
$$Q_h := \{q_h \in S^{p_1-1,p_2-1,p_3-1}_{\boldsymbol{\alpha_1-1,\alpha_2-1,\alpha_3-1}}\}$$

Collocation Scheme

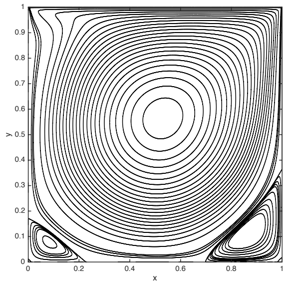
- Rotational form of steady Navier-Stokes:

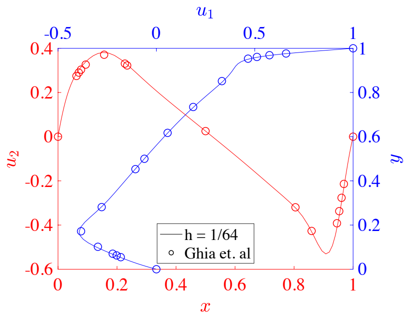
$$\nu \nabla \times \boldsymbol{\omega} + \boldsymbol{\omega} \times \mathbf{u} + \nabla P = \mathbf{f}$$
$$\nabla \cdot \mathbf{u} = 0$$
$$\boldsymbol{\omega} - \nabla \times \mathbf{u} = 0$$

- No penetration BCs are enforced strongly
- No slip BCs are enforced weakly by adding a suitable penalty term to constitutive law
- Collocate at Greville abscissae for each space



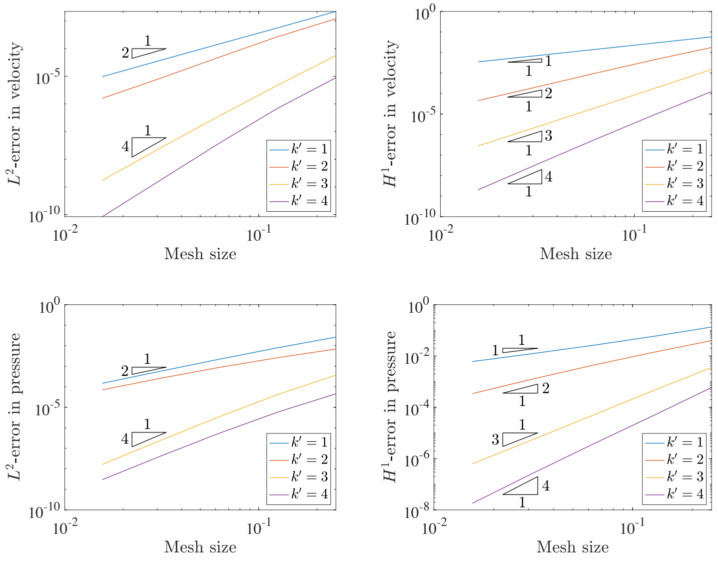
Re = 1000 Lid Driven Cavity





- Results match well with established reference data across Re

Manufactured Vortex Solution



- Convergence rate of solution is optimal for odd degrees, 1 degree suboptimal for even degrees
- Convergence rate of derivative is optimal for all degrees

J. A. Evans and T.J.R Hughes. "Isogeometric divergence-conforming B-splines for the steady Navier–Stokes equations." *Mathematical Models and Methods in Applied Sciences* 23.08 (2013): 1421-1478.