Synthetic Principle Compenent Design

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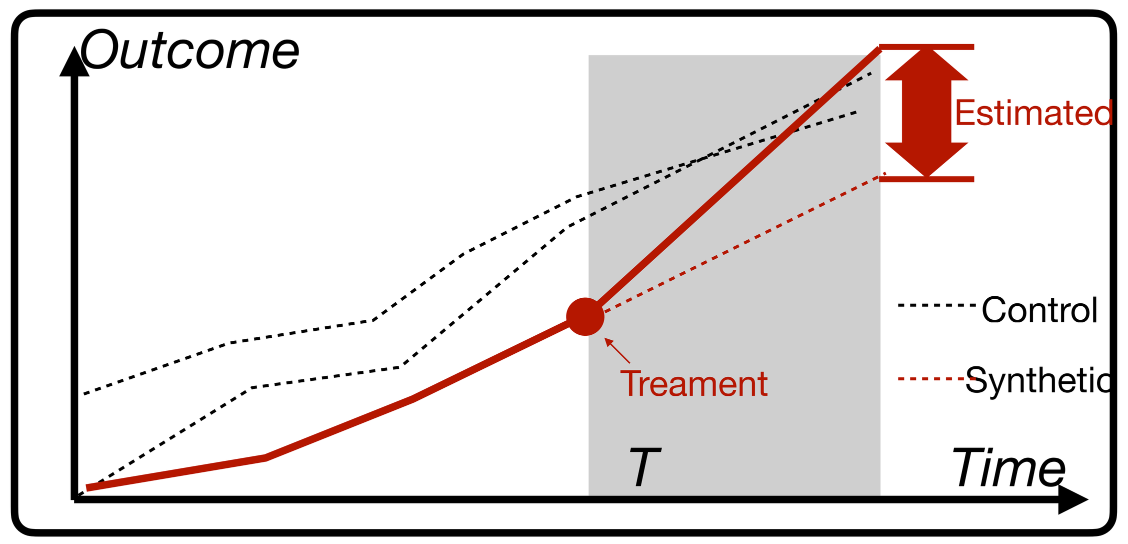
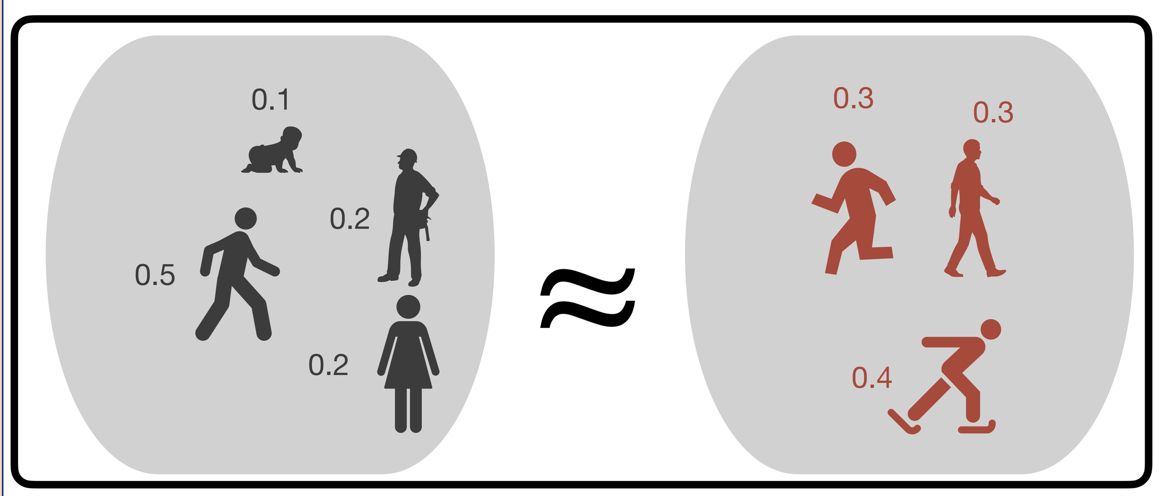
Problem Introduction

Synthetic Control: estimate the treatment effect for panel data. What's the counterfactual outcome?
Synthetic (Regression) from control group.



California = 0.334* Utah+0.234*Nevada+...

What if I can decide who to treat



Match weighted average

$$\min_{\{D_i,w_i\}_{i=1}^N} \frac{1}{T}\sum_{t=1}^T\left(\sum_{i=1}^N w_iD_iY_{it} - \sum_{i=1}^N w_i(1-D_i)Y_{it}\right)^2 + \lambda\sum_{i=1}^N w_i^2$$

s.t. $w_i \geq 0,\quad D_i \in \{0,1\}\ \text{for } i=1,\dots,N,$

$$\sum_{i=1}^N D_i = K,\quad \sum_{i=1}^N w_iD_i = 1,\quad \sum_{i=1}^N w_i(1-D_i) = 1$$

Droped in our paper

Min $\|Y(w\odot d)\|_2^2$

s.t. $\|w\odot d\|_1 = 1, 1^\top(w\odot d) = 0$

Penalize the constraint

Min $(w\odot d)^\top(Y^\top Y + \lambda 11^\top)(w\odot d)$

s.t. $\|w\odot d\|_1 = 1$

Gives the right sign (experiment)

Equal to Phase Retrieval

Thm

Equivalent to

Inverse covariance matrix

$$\max_{y\in\{-1,1\}} \|(Y^\top Y + \lambda 11^\top)^{-1}y\|$$

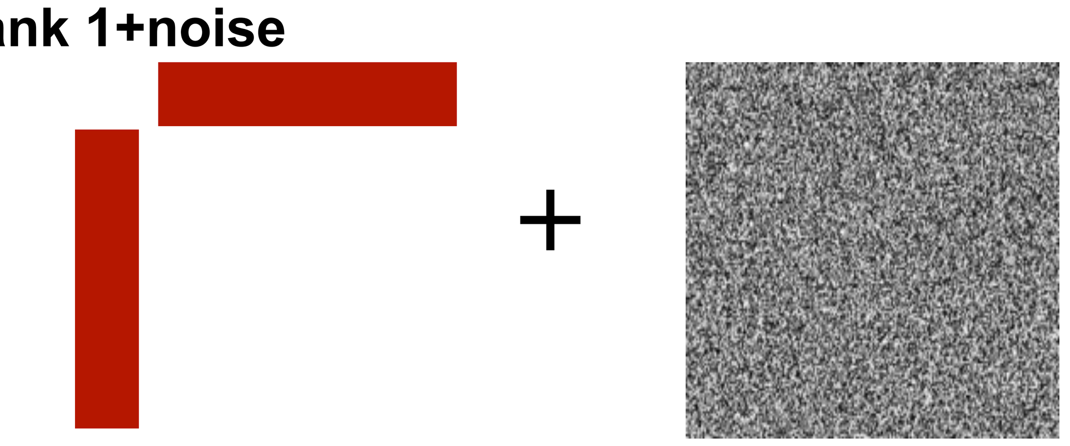
Find phase

Phase Synchronization: find a phase of a complex signal when the covariance matrix is known, Still NP-hard.

Treat Unit i if y(i)=1
Put Unit i in control group, if y(i)=-1

Theory

Rank 1+noise



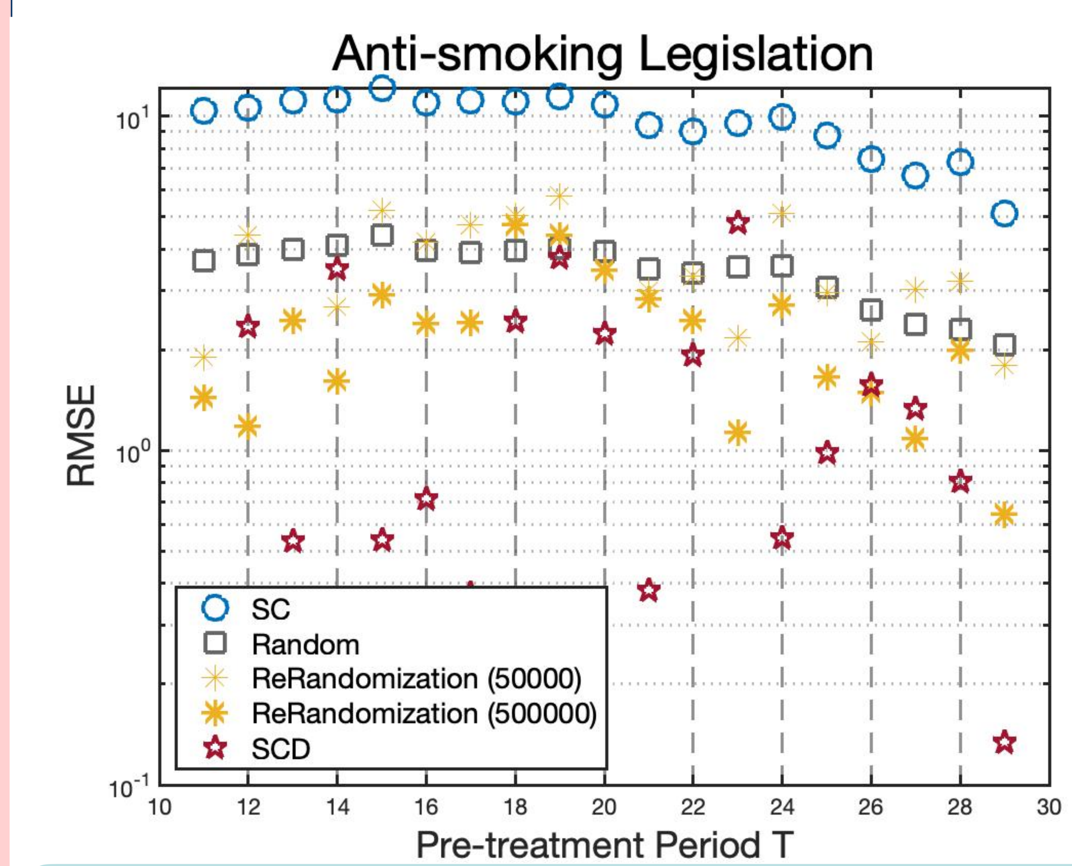
Globally solvable in this random data generating process. What does this means for Synthetic Control?

Inverse covariance matrix is rank 1
Only one feasible (error=0) experiment design for you data !

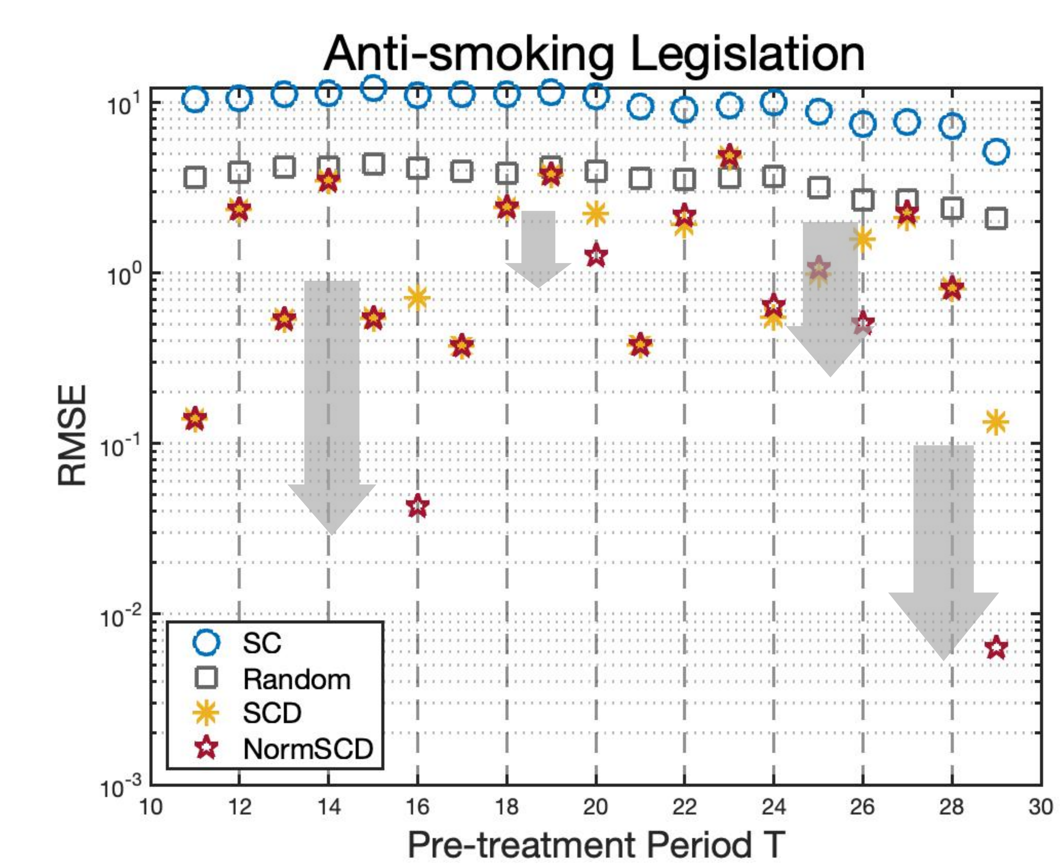
Fast experiment design method for Causal Panel Data via Simple Inverse Power Method

Numerical Results

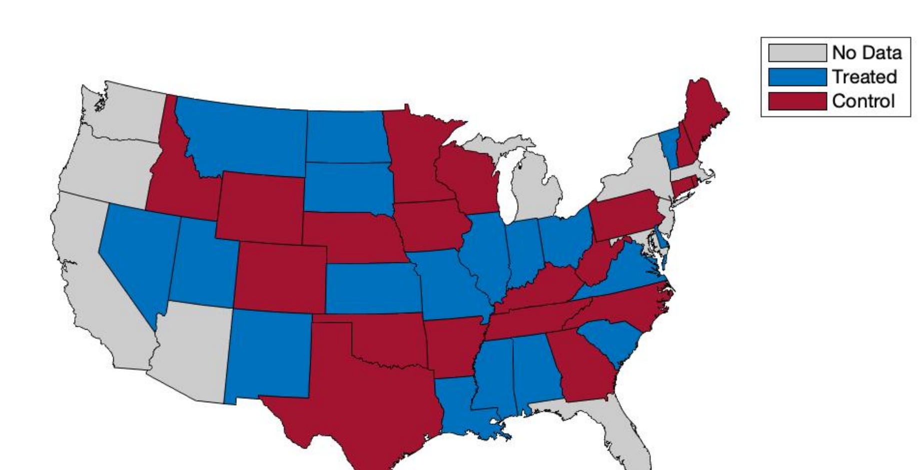
Dataset: The Abadie–Diamond–Hainmueller Smoking Data
More simulation see paper.



Better than 500,000 rerandomization



Normalization Always Helps!



Spectral Algorithm

Spectral Initialization:

$$\max_{\|y\|_2=N} \|(Y^\top Y + \lambda 11^\top)^{-1}y\|$$

Smallest eigenvector of the covariance matrix is a good guess!

Generalized Power Method:

Power Method

$$y_{k+1} = \text{sgn}[(Y^\top Y + \lambda 11^\top)^{-1}y_k]$$

Once knows the experiment profile y, solving the optimal weight w is a **convex** optimization method, which is simple!

Normalized Variate

Theory for Phase Synchronization needs data generated from binary signal, however our data is not binary (have a weight w)

Idea: Normalize!

$$y_{k+1} = \text{sgn}[(Y^\top Y + \lambda 11^\top)^{-1}y_k./d]$$

We normalize the matrix via its diagonal entries (which is a good guess of the weight magnitude).

$$d = \sqrt{\text{diag}((Y^\top Y + \lambda 11^\top)^{-1})}$$

Weaker assumptions for global convergence!