Online Learning for Traffic Routing under Unknown Preferences

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Introduction and Motivation

- Road tolls help cope with efficiency losses due to selfish user routing by users to minimize individual travel costs.
- However, the efficacy of road tolling schemes relies on having access to complete information on users’ trip attributes, e.g., their values of times and O-D pairs.
- We propose an online learning approach to set tolls that preserves user privacy.

For the same tolls, the resulting traffic flows will be different if the value of time distribution of users is different.

System Optimization

Minimize Total Travel Cost while satisfying Road capacity constraints

\[
\begin{align*}
\min_{f, \lambda} & \quad U^* = \sum_{u \in U} \left( \nu_u \sum_{P \in P_u} l_P f_{P,U} + \lambda_u f_{U,U} \right) \quad \text{Total Travel Cost} \\
\text{s.t.} & \quad \sum_{P \in P_u} f_{P,U} + f_{U,U} = 1, \quad \forall u \in U, \quad \text{Allocation Constraints} \\
& \quad f_{P,U} \geq 0, f_{U,U} \geq 0, \quad \forall P \in P_u, u \in U \\
& \quad \sum_{u \in U} \sum_{P \in P} f_{P,U} \leq c_e, \quad \forall e \in E, \quad \text{Capacity Constraints}
\end{align*}
\]

Optimal Tolls can be set through the dual variables of the capacity constraints of the above problem, but this requires complete information on users’ values of time and O-D pairs.

Online Learning Algorithm

Privacy-Preserving Tolling Algorithm: Only relies on observed edge flows and not on any user attributes

Algorithm 1: Efficient Routing via Privacy-Preserving Tolls

1. Time Period \( T \), Road Capacities \( c \)

2. Set the Toll \( \tau^{(1)} \) to \( 0 \)

3. For \( t = 1, \ldots, T \)
   - Phase I: User Equilibrium for Toll \( \tau^{(t)} \)
     - Observe Edge Flows \( \mathbf{x} \)
   - Phase II: Toll Update
     - \( \tau^{(t+1)} = (\lambda_t - \gamma(c - \mathbf{x}))_+ \)

4. End

Performance Metrics

- Regret
  - Optimality gap between an online allocation and an offline oracle with complete information on users’ preferences
- Capacity Violation
  - Norm of cumulative excess flow beyond each edge’s capacity

Theorem (Lower Bound): Under i.i.d. user arrival, the regret of any algorithm is \( \Omega(\sqrt{T}) \), where \( T \) is the number of time periods.

Theorem (Upper Bound):
- Under i.i.d. user arrival, the regret and constraint violation of Algorithm 1 are \( O(\sqrt{T}) \), where \( T \) is the number of time periods.

Future Work

1. Developing regret and constraint violation guarantees when users’ trip attributes are not i.i.d.
2. Extending results to setting when travel times vary as a function of the flows.
3. Extending to objectives beyond system efficiency, e.g., fairness or revenue.