The Price of Simplicity in the Stationary Prophet Inequality Problem

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Model

- Infinite horizon, continuous-time marketplace
- One seller of a single good, items of which are supplied & perish according to a Poisson process with rates $\lambda > 0, \mu = 1$
- n buyer types, where buyer type i arrives according to an independent Poisson process with rate $\gamma_i > 0$
- When buyer of type i arrives, he bids a fixed value v_i for the good and seller makes an irrevocable decision to either sell & collect v_i or to reject
- **Objective:** maximize seller's expected average revenue

Motivation: Optimal vs. Simple

Optimal Policy

- Generates maximum expected average revenue
- Lacks economic interpretation
- Potentially impractical to implement

"Simple" Policy

- Sub-optimal from revenue perspective?
- Strong economic interpretation
- Straightforward to implement

Fixed-threshold policy: seller sets a minimum price $\bar{p}$ he is willing to accept for the good (sell iff $v_i \geq \bar{p}$)

Main Results

A new benchmark (UB):

$$\begin{aligned} \max \sum_i v_i \cdot x_i & \\ \text{s.t.} \sum_i x_i \leq \lambda & \\ x_i \leq \gamma_i \cdot (1 - \exp(-\lambda)) & \\ x_i \geq 0 & \end{aligned}$$

A fixed-threshold policy (FT):

let x^* be an optimal solution to UB
for arrival of buyer of type i
if item available
sell to buyer w.p. $p_i \triangleq \frac{x_i^*}{\gamma_i \cdot (1-\exp(-\lambda))}$

Theorem 1: Fixed-threshold policy FT is $1/2$-competitive with the optimal offline policy.

Corollary: Inventory capacity of just 2 items is sufficient to guarantee FT is still $1/2$-competitive.

Theorem 2: No online policy is better than $1/2$-competitive with the optimal offline policy.

Proof Strategies

Theorem 1:

1. Prove $UB \geq OPT_{\text{offline}}$
2. Prove $FT \geq 1/2 \cdot UB$ ()

(1) + (2) $\Longrightarrow FT \geq 1/2 \cdot OPT_{\text{offline}}$

Theorem 2:

Two buyer types:

- Rare big spender: $\gamma_1 = \epsilon, v_1 = 1 + \frac{1}{\epsilon}$
- Common miser: $\gamma_2 = \infty, v_2 = 1$

Prove $OPT_{\text{online}} \leq 1/2 \cdot OPT_{\text{offline}}$

Techniques: queuing theory, continuous time Markov chain analysis, stochastic dominance

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